Compressed Measurement of Structural Similarity Index

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Compressed Measurement of Structural Similarity Index

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ABSTRACT

Image quality index corresponds to amount of degradation in an image. Structural Similarity Index(SSIM) is one such image quality index considered under this project. The statistical parameters required for the computation of SSIM are defined in spatial domain. The computations involved can be reduced when performed in compressed or reduced domain. In addition, as many image processing applications are based on compressed data, reconstructing inferior quality image for quality measurement can also be avoided. Compressed data can be achieved either at processing level (image compression algorithms) or at acquisition level (compressive sampling). A new algorithm called CM-SSIM was proposed to compute the SSIM from the compressed data. This is done through reconstructing data from reduced dimension to appropriate basis system and defining statistical parameters associated with the basis system. The proposed algorithm is validated by performing correlation analysis between the DMOS with acutal SSIM and CM-SSIM. The correlation factors considered for analysis are pearson, spearman and kendall tau. The results confirms that the proposed algorithm exist a good relationship with the DMOS & actual SSIM and would be suitable for future real-time systems.

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Sankarasrinivasan S

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Abbreviations

SSIM	Structural Similarity Index
CS	Compressive Sampling
CM	Compressed Measurement
DFT	Discrete Fourier Transform
DCT	Discrete Cosine Transform
DWT	Discrete Wavelet Transform
DMOS	Degradation Mean Opinion Score
MSE	Mean Square Error
HVS	Human Visual System
FRIQ	Full Reference Image Quality Index
RRIQ	Reduced Reference Image Quality Index
NRIQ	No Reference Image Quality Index
MATLAB	Matrix Laboratory
GUI	Graphical User Interface
RIP	Restricted Isometric Property
IID	Independent and Identically Distributed
ICA	Image Compression Algorithms
DMD	Digital Micromirror Device

Dedicated to my parents, brother and all my friends

1 Introduction

High resolution image can be effectively preserved with relatively small number of coefficients when represented on an appropriate basis. This representation gives reduction in storage, processing and transmission. Since natural images are compressible in appropriate basis, sampling scene with millions of pixels to obtain HD images and then compressing it, results in ineffective use of sensors.In general, Sampling images/signals follow Shannon theorem which states that sampling rate must be at least twice the bandwidth of the signal. Images are not naturally band limited and its sampling rate is decided by spatial resolution. However acquisition system use anti-aliasing low pass filter before sampling to band limit it, where Shannon theorem plays a crucial role.

In recent times a theory has been proposed called,Compressive sampling which asserts that certain signal can be recovered from far few samples than suggested by Nyquist rate. The criteria are signal should be compressible in a certain transform domain and sampling method has to satisfy property of incoherence with the transform used. This concept was adopted in this work, for the computation of image quality index.

Representation in Compressed Domain

Data compression is possible either at the sensing level or at processing level. The data can be sampled completely and then concisely represented in appropriate space(Image Compression Algorithms). Otherwise, it can be directly sensed in the compressed form(CS theory). When represented in the reduced dimension, whether a data suffers any information loss or not, have to be analyzed by some experiments.

In this experiment an image is taken and transformed to Discrete Cosine Transform (DCT) domain. The reconstruction is done by taking only few percent of the total coefficients. SSIM was computed between the actual and the reconstructed image. The result shows that, image reconstructed with upto five percent of the coefficients, have good visual quality.

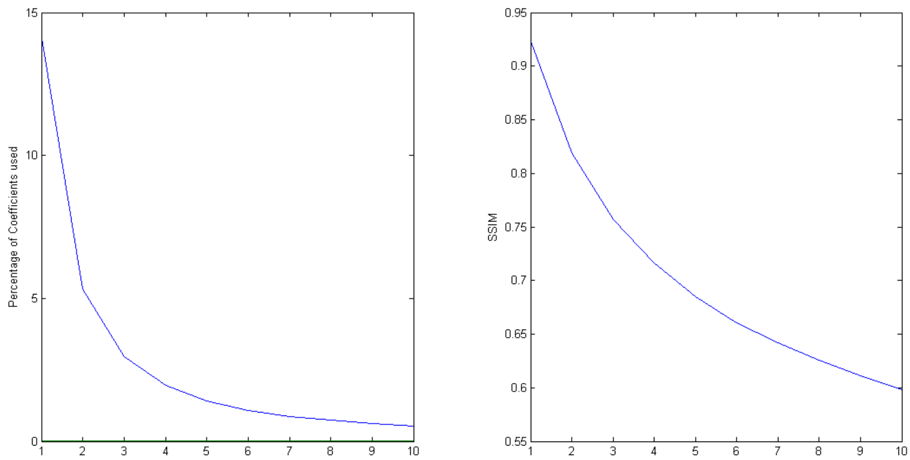


FIGURE 1.1: DCT Coefficients and SSIM

In the next experiment, a sparse signal was taken and represented in the reduced dimension(by CS theory, which is approximately one fourth of its actual dimension). The mean square error was computed between the actual and the reconstructed signal. This experiments was repeated several times and the box plot corresponding to MSE was plotted.

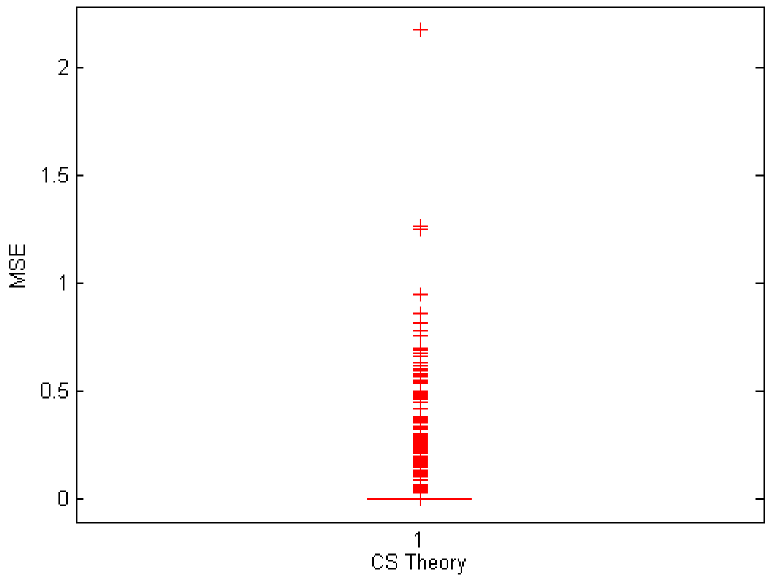


FIGURE 1.2: MSE on Reconstructed Signal

From the box plot, it can be observed that most of the MSE obtained are of value zero. This ensures that projection of data into a reduced dimension and retrieving it back, doesn't affect the nature of the data.

The fact that signal when represented in reduced/compressed domain can be exactly reconstructed without any loss of information is adopted in the compressed measurement of quality index.

1.1 Literature Review

The literature survey has been mainly focused on compressive sensing theory and its emerging techniques. The following are some of the papers referred for the literature survey.

In this paper[8], the author provides a new approach to develop a structure based image quality index. The problem associated with the existing error visibility methods, necessity of human visual system(HVS) based metrics and detailed mathematical derivation for structural similarity index were given. The

Table 1.1: Literature Survey

Domain	Authors	Highlights	
		Advantages	Disadvantages
Compressive Sampling	E. Candes and M. Wakin[1]	Introduction to CS	Practical complexities not mentioned
	Duarte-Carvajalino J.M. and Sapiro G[2]	Joint optimization algorithm	Works for trained data sets.
	Mark A. Davenport, Michael B. Walkin and Richard G. Baraniuk[3]	Procedure for learning in compressed domain.	No details on setting parameters
	Jingming Sun, Shu Wang and Yan Dong[5]	Development of Structured sampling matrix	Require more number of measurements
SSIM	Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli[8]	Complete explanation with mathematical support	Usage of some constants are undefined
Mathematical Modelling of SSIM	Sumohana S. Channappayya, Alan Conrad Bovik[13] , and Robert W. Heath	Mathematical proof for SSIM in DCT domain	Range of the constants are undefined.
	Xuejun Dong and Haitao Li[6]	Statistical analysis of wavelets	Mathematical explanations are not elaborate

experimental results shows that the proposed index exhibit perfect relationship with the objective score (MOS).

The author of this paper[1] has provided a basic introductory part to compressive sampling theory. A new sampling technique was proposed and theoretically proven to be effective than the classical Nyquist-Shannon sampling method. Few practical applications are mentioned, but constraints involved in hardware implementation are not considered. In Overall, even though it didn't provide the complete idea, the essence of the whole theory is clearly defined.

In this paper [5], author provides a new structure based sampling matrix called sparse block circulant matrix to reduce hardware and computation complexities. Even though i.i.d. Gaussian and Bernoulli matrix satisfy restricted isometric property and aids perfect reconstruction, its high unstructured nature makes both the projection and reconstruction expensive. Structured sampling matrices are practically more economical and involves less complicated hardware. The simulation result validates SBCM reduce the computational burden but keeps the similar reconstruction accuracy as random matrices in signal reconstruction.

In this paper[3], author provides theoretical results for learning in the compressed domain. This report focuses on the detection and estimation problem in the compressed domain. The author demonstrate procedures to solve for a variety of signal detection and estimation problem given with the compressed data without going for its exact reconstruction. The application of compressive sampling in statistical inference tasks are also mentioned.

This paper[2] elaborate on emerging techniques in the field of compressive sampling especially in optimizing sampling matrix. A framework was introduced for the joint design and optimization of over-complete dictionary along with the sensing matrix to a set of training images. The results shows that joint optimization outperforms existing techniques which are based on independent optimization.

In this article[16], author proposes a new approach to develop a simple, compact and economical digital cameras based on compressive sampling. As this new flexible architecture captures data directly in the compressed form, it is suitable and cost efficient for HD image/video applications.

In the theory of compressive sampling, maximal incoherence is achieved by taking random measurement matrices. But structured matrices like toeplitz and circulant are practically more realizable. This paper[17]introduces fast algorithms to recover the signal from the incomplete structured matrices. Experimental results validates that structured matrices are fast and efficient for signal encoding and decoding.

1.2 Motivation of the Project

High definition image/video transmission demands huge amount of data to be transmitted. As transmission channels have restricted bandwidth, the data compression became inevitable. The forward transform make use of convenient basis (like Discrete Fourier Transform(DFT), Discrete Cosine Transform(DCT) and Discrete Wavelet Transform(DWT)) to represent data with few number of coefficients and compressed data was transmitted. In the receiver end, inverse transform was done to restore the original signal. The SSIM was actually defined in the spatial domain, and if we mathematically model the required parameters(for SSIM)in frequency domain itself (i.e. Fourier, DCT or wavelets etc.) then for the received inferior quality image, doing inverse transform can be avoided.

In emerging sensing applications, the demanding Nyquist rate is so high that we end up in far too samples[19]. And realizing such systems requires huge amount of sensors and memory, which is practically costlier and complicated. Due to the advent of compressive sampling, we may soon expect new sensing architectures[16] which are capable of obtaining the compressed data. This technique reduce memory requirements, but involves complex circuitry. The CS sensing procedure follows randomized sampling and smart decoding techniques which pose a lot of challenges in analyzing and reconstructing the compressed data.

The idea to find image quality index given with the compressed data is discussed in this thesis.

1.3 Objective

- Mathematical modelling of SSIM in different dictionaries like DFT, DCT and Wavelets.
- Understand and implement the concept of Compressive Sampling
- Analysis and statistical support for the proposed CM SSIM

1.4 Thesis Organisation

- Chapter 1: Introduction
- Chapter 2: Structural Similarity Index
- Chapter 3: Compressive Sampling
- Chapter 4: Compressed Measurement of SSIM
- Chapter 5: Results & Discussion

2 Structural Similarity Index

2.1 Image Quality Index

Images are subjected to various type of distortion during acquisition, processing, transmission and so on. Depending upon the nature of degradation, the visual quality may get affected in a different way. For quality assurance, image quality assessment has to be done before being presented to the user.

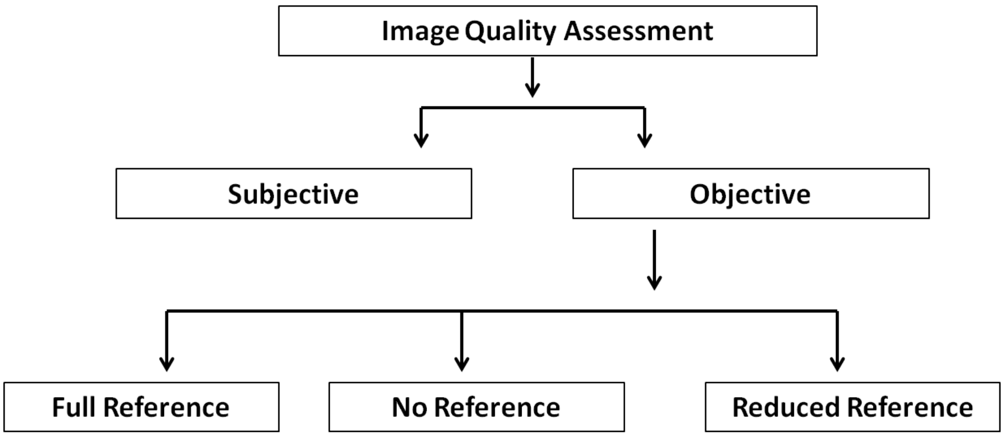


FIGURE 2.1: Image Quality Assessment

Image quality assessment may be either subjective or objective. Under subjective analysis, an image and its different distorted versions were presented to the users for rating and quality score was assigned. Even though this method is reliable, the process involved are inconvenient and time consuming. In order to overcome this, we need to go for the algorithms which are capable of analyzing images like a human eye (i.e. objective algorithms).

The formulation of objective algorithms are based on the statistical parameters associated with an image. Under objective analysis, depends on the availability of the reference image they are classified as full reference image quality index (FRIQ), reduced reference image quality index (RRIQ) and no reference image quality index (NRIQ). Under this thesis, the FRIQ based objective algorithm is adapted.In FRIQ algorithm a perfect version of an image is being compared against its distorted version. In general, the perfect versions are obtained from high quality acquisition devices and need more resources than the distorted versions.

2.2 Structural Similarity Index

Structural Similarty Index is a FRIQ based algorithm. The quality index was computed based on the degradation of structure between the two images. This method have a good relationship with the subjective quality score.

Analysis between SSIM and MSE

The traditional full reference(FR) methods are mean square error and PSNR. Though, they are all simple and mathematically convenient, its quality estimation does not match well with humans. An experiment was conducted to gain a better understanding over this issue. Different distorted versions of an image was taken and compared with reference image, both MSE and SSIM were computed.

From the results it is obvious that for the same value of MSE, their exist different SSIM value. The reasons is MSE measures the amount of error introduced whereas SSIM computes quality based on type of error introduced. For the same

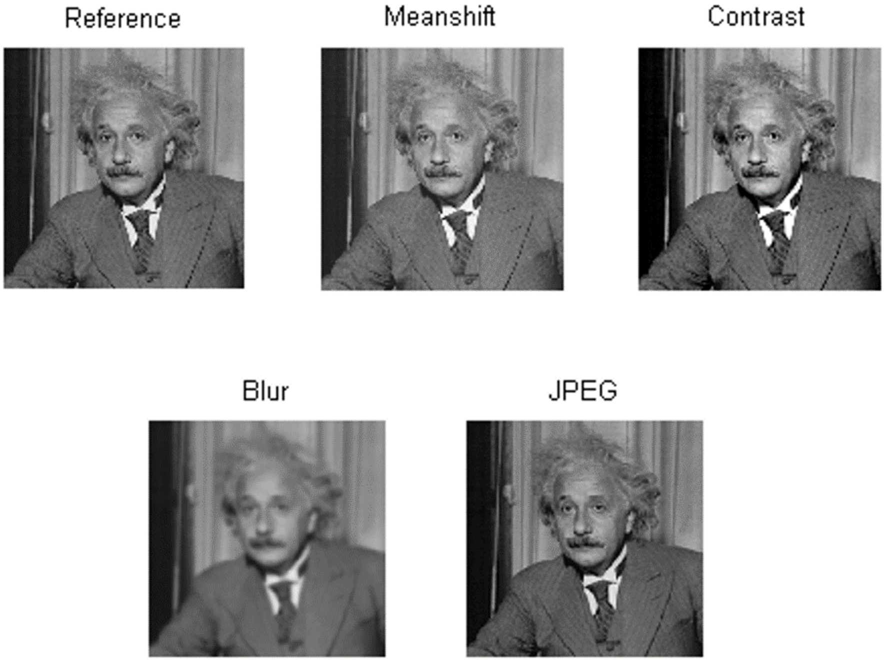


FIGURE 2.2: Images taken for compartive analysis between SSIM and MSE

TABLE 2.1: Experimental Results (SSIM and MSE)

S.No	MSE	SSIM
Meanshift	144.22	0.9884
Contrast	144.22	0.9133
Blur	143.90	0.6940
JPEG	141.59	0.8697

amount of error depending upon its nature, visual quality may get affected in a different way.

And a new paradigm of quality measure called similarity measure was developed, based on human visual system and proved to have better correlation with the perceived quality measure.

2.3 Derivation of SSIM

The luminance of an object is given by the product of illumination and reflectance. Usually the structure of an object is independent of its illumination. So in order to explore the structural similarity measure, effect of illuminance have to be separated out from an image. This is done by representing image with attributes that are independent of average luminance and contrast.

Assume x and y be the two non negative images. Under this scheme, the similarity measure is based on three components: mean, variance and structure. Overall similarity index is given by

$$S(x,y) = f(l(x,y), c(x,y), s(x,y)) \tag{2.1}$$

where x,y corresponds to reference and test image
l(x,y)=luminacne; c(x,y)=contrast; s(z,y)=contrast;

The luminance is defined interms of mean intensity of an image.

$$\mu = 1/N \sum_{i=o}^{N-1} x_i \tag{2.2}$$

and luminance comparison is a function of μ_x and μy

$$l(x,y) = \frac{2\mu_x\mu_y + c1}{(\mu_x^2 + \mu_y^2 + c1)} \tag{2.3}$$

The effect of luminance can be eliminated by subtracting the mean term, and the contrast comparison is a function of standard deviation σ_x and σ_y

$$c(x,y) = \frac{(2\sigma_x\sigma_y + c2)}{(\sigma_x^2 + \sigma_y^2 + c2)} \tag{2.4}$$

In the above equations, $c1 = (K_1 * L)^2$; $c2 = (K_2 * L)^2$;
L-dynamic range of the pixel value;
$K_1 \ll 1$and $K_2 \ll 1$ are small constants.

Finally structural comparison is carried out, after normalizing x and y by its own standard deviation. i.e. (x-μ_x)$/\sigma_x$ and is given by

$$S(x,y) = \frac{\sigma_{xy} + c3}{(\sigma_x\sigma_y + c3)} \tag{2.5}$$

Overall formula for the Structural similarity index is given by combining all the above the parameters equations.

$$s(x, y) = l(x, y)^{\alpha} c(x, y)^{\beta} s(x, y)^{\gamma} \tag{2.6}$$

here α ,β and γ are kept to adjust the importance of three parameters. (where $\alpha \geq 0; \beta \geq 0; \gamma \geq 0$);

In order to further simplify the equation, α ,β and γ were taken as unity and value of c3=c2/2;

$$s(x, y) = \frac{(2\mu_x \mu_y + c1)}{(\mu_x^2 + \mu_y^2 + c1)} \frac{(2\sigma_{xy} + c2)}{(\sigma_x^2 + \sigma_y^2 + c2)} \tag{2.7}$$

The values of the constants used are as follows,

- K1=0.01
- K2=0.03
- L=255 (for gray scale image)

Properties of SSIM

1. It is symmetric; S(x; y) = S(y; x)
2. The index value stays within unity; S(x; y) ≤ 1
3. Incase, if x and y are equal; S(x; y) = 1

2.4 SSIM Formulation In Different Dictionaries

The statistical parameters associated with the SSIM are mean, variance and co-variance. When the image is transformed to the representation domain, the nature of distribution changes depends on dictionary basis system. Hence, the spacial domain mean, variance and covariance required for the computation of SSIM have to defined in DFT, DCT and Wavelet domain.

It is a fact when an image is represented in appropriate basis system, the energy associated with it doesn't change. This is given by persevals theorem which asserts that energy will get preserved in both the domain. The energy conservation theorem between spatial and frequency domain is given by

$$\sum_{n=0}^{N-1} |x[n]|^2 = 1/N \sum_{k=0}^{N-1} |X(k)|^2 \tag{2.8}$$

This property is applicable to domains which are represented by basis. Basis are set of vectors which are orthogonal or orthonormal to each other. All the transforms considered here, satisfy this property(i.e. unitary transforms).

2.4.1 Equivalent in DFT Domain

The fourier transform of a function is given by

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi i k n/N} \tag{2.9}$$

DFT is an unitary transform and obeys the parseval theorem. Using this property and above equation, the relation between spatial components in terms of DFT coefficients can be found out.

Equation for Mean, Variance and Covariance are as follows

$$\mu_x = \frac{\sum_{i=0}^{n-1} x(i)}{N} = \frac{X(0)}{(N)} \tag{2.10}$$

$$\sigma_x^2 = \frac{\sum_{k=1}^{N-1} X(k)^2}{(N-1)^2} \tag{2.11}$$

$$\sigma_{xy} = \frac{\sum_{k=1}^{N-1} X(k)Y(k)}{(N-1)^2} \tag{2.12}$$

2.4.2 Equivalent in DCT Domain

In order to find SSIM from DCT coefficients, the spatial domain mean, variance and covariance should be expressed in terms of DCT coefficients.

DCT of a vector x ϵ R^N

$$X(k) = \sum_{i=0}^{n} \lambda(k) cos \frac{(2i+1)\pi k}{2N} x(i); \tag{2.13}$$

DCT is a unitary transform and obeys the parseval theorem. Using this property and above equation, the relation between spatial components in terms of DCT coefficients can be found out.

Equation for Mean, Variance and Covariance are as follows

$$\mu_x = \frac{\sum_{i=0}^{n-1} x(i)}{N} = \frac{X(0)}{\sqrt{(N)}} \tag{2.14}$$

$$\sigma_x^2 = \frac{\sum_{k=1}^{N-1} X(k)^2}{N-1} \tag{2.15}$$

$$\sigma_{xy} = \frac{\sum_{k=1}^{N-1} X(k)Y(k)}{N-1} \tag{2.16}$$

2.4.3 Equivalent in Wavelet Domain

Wavelets are the functions of finite duration and varying frequency. More precisely, it consists of waves which begins at zero, increases and then decreases back to zero. In wavelet transform, the data function is convoluted with the shifted and stretched versions of a wavelet function along a given time or space range. There are varieties of wavelets functions are available and were chosen depends on application. In this work, Haar wavelet is chosen for the mathematical modelling. The Haar Wavelet is given by the function

$$f(n) = \begin{cases} +1 & \text{if } 0 \leq x \leq (1/2) \\ -1 & \text{if } (1/2) \leq x \leq 1 \end{cases}$$

Applying the same for the Haar Wavelet(Refer [6] & [7])

$$\sum_{n=0}^{N-1} |x[n]|^2 = 1/N \left(\sum_{j=1}^{N/2^J} |S_k|^2 + \sum_{k=1}^{J} \sum_{j=1}^{N/2^k} |d_{j,k}|^2 \right) \tag{2.17}$$

here d(j,k)corresponds to the jth coefficient at the kth level of decomposition. S(k)corresponds to kth coefficient following J levels of decomposition.

TABLE 2.2: SSIM in Different Dictionary

Parameters	DWT	DCT	DFT
Mean	1/N $\sum_{j=1}^{N/2^j} \lvert S_k\rvert^2$	$\mu_x = \frac{\sum_{i=0}^{n-1} x(i)}{N} = \frac{X(0)}{\sqrt(N)}$	$\mu_x = \frac{\sum_{i=0}^{n-1} x(i)}{N} = \frac{X(0)}{(N)}$
Variance	$\sum_{k=1}^{J} \sum_{j=1}^{N/2^j} \lvert d_{j,k}\rvert^2$	$\sigma_x^2 = \frac{\sum_{k=1}^{N-1} X(k)^2}{N-1}$	$\sigma_x^2 = \frac{\sum_{k=1}^{N-1} X(k)^2}{N-1^2}$
Covariance	$\sum_{k=1}^{J} \sum_{j=1}^{N/2^k} \lvert dx_{j,k} dy_{j,k}\rvert$	$\sigma_{xy} = \frac{\sum_{k=1}^{N-1} X(k)Y(k)}{N-1}$	$\sigma_{xy} = \frac{\sum_{k=1}^{N-1} X(k)Y(k)}{N-1^2}$

3 Compressive Sampling

3.1 Introduction

In general, image acquisition system follows Shannon-Nyquist theorem i.e. sampling rate must be at least twice the bandwidth of the signal. Even though images are not band limited, while acquisition anti-aliasing low pass filters are used before sampling and hence Shannon-Nyquist theorem plays an implicit role.

In recent times a theory has been proposed called,Compressive sampling which asserts that certain signal can be recovered from far few samples than suggested by Nyquist rate. Compressive sampling achieve dimensionality reduction at the acquisition level i.e. the acquired data is directly obtained in the compressed form. The criteria are signal should be compressible in a certain transform domain and sampling method has to satisfy property of incoherence with the transform used. Implementation of CS theory further reduces our time and memory complexities.

3.2 Mathematical Background

Any problem can be summarized in the form

$$A_{m\times n} * X_{n\times 1} = Y_{m\times 1} \tag{3.1}$$

Depending on nature of transformation A may be of any of the following

- Square Matrix
- Under determined form
- Over determined form

In compressive sampling, we deal with under determined set of linear equation, i.e. too few equation and too many unknown. This set has infinite number of solutions. In order to solve it, some constraints have to be introduced. In case of CS, solvers look for the sparsest solution set.

3.3 Sparsity and Incoherence

In order to apply compressive sampling[18], a signal has to be sparse in appropriate basis and the sampling protocol has to satisfy the property of incoherence with the sampling matrix.

- Sparsity
- Incoherence

Sparsity A signal is said to be sparse, if it has few number of non-zero coefficients over a large space. Any signal can be made into sparse form, by representing them in the appropriate basis. The dictionaries or basis which were chosen should have energy compaction and de-correlation property thereby it takes of the redundancy information and represent the signal concisely.

Let x ϵ R^N be an N pixels image ;
ψ= be an orthonormal basis

$$x = \sum_{i=1}^{N} f_i \psi_i(t) \tag{3.2}$$

here f is the coefficient of x in the basis ψ.A image is said to be sparse if most of the coefficients of are zero. Natural images are sparse when represented on an appropriate basis like DFT, DCT and Wavelets

Incoherence The condition is sampling matrix should be as different from our representation basis. Coherence measures the similarity between the two basis. Lower value indicates smaller coherence and therefore the basis are maximally incoherent i.e. the signal which is sparse in the representation basis have to be dense in the sampling space. Lower coherence measurement matrix are preferred to reduce the number the measurements required.

CS theory requires sensing matrix and representation matrix should be maximally incoherent.

Φ = [Φ_1 Φ_2 ..... Φ_N] be an mxN sampling matrix

Ψ be a sparse representation matrix

Coherence is given by ϕ and ψ is given by

$$\mu(\psi, \phi) = sqrt(N).max_{1 \leq k, j \leq n} | < \psi_k, \phi_j > | \tag{3.3}$$

3.4 Sampling Matrix

The role of the sampling matrix is very significant. It should be capable of representing, say an N signal dimensional signal in a much reduced dimension without losing the essential information present i.e. it should aid in perfect reconstruction of the signal.

The necessary condition that sampling matrix has to satisfy is restricted isometric property and null space property [20]. This properties ensures that the signal can be represented within the column space of the total solution space; thereby the reduction in dimension can be achieved by ignoring the information present in the null space. This mathematical expression for RIP is given by

$$1 - \delta \leq \frac{||\phi x_2 - \phi x_1||}{||x_2 - x_1||} \leq 1 + \delta \tag{3.4}$$

(1-δ) and (1+δ) are the lower and upper bounds. Satisfying the condition ensures that in the compressed domain energy stays within bounded limits. (where δ is a small value)

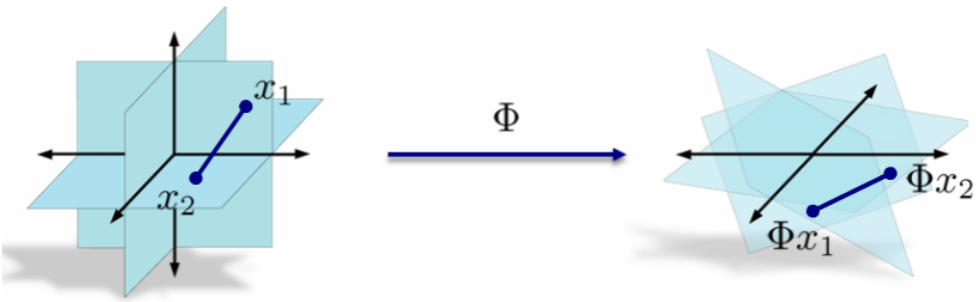


FIGURE 3.1: Restricted Isometric Property[19]

In equation 3.4, l2_norm was applied between the reduced and the actual signal. It checks whether the signal stay within some restricted dimension or not.

An experiment was conducted to check for the condition in equation 3.4 for random matrices. An N dimensional signal is projected into a M dimension system by means of random gaussian measurement matrix. The L2-norm was computed between the reduced and actual signal. The experiment was repeated for several times.

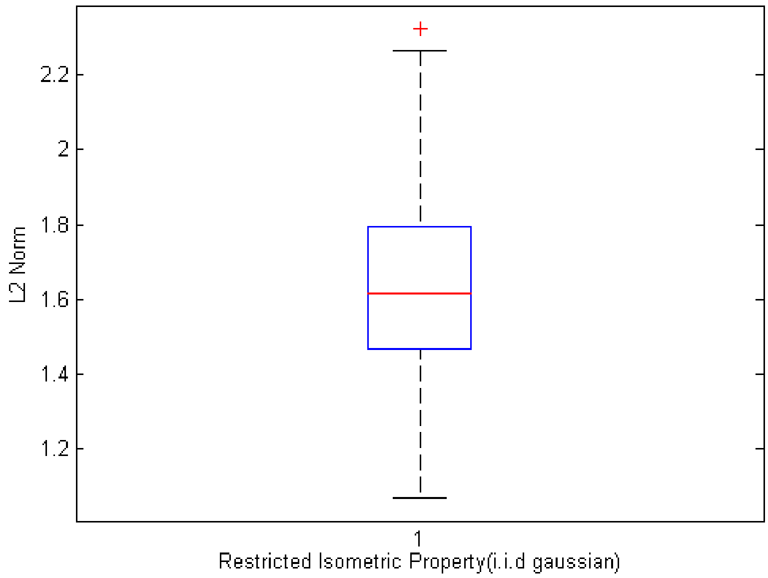


FIGURE 3.2: L2 Norm Computation

Observing the results it is clear that norm value obtained are close to one. This assures that the random measurement system preserves the dimension of a vector when projected to a lower dimension.

Reduced Dimension CS ensures that if the number of measurements(m) satisfies the below condition, then reconstruction is exact.

$$m \geq C * \mu^2(\psi, \phi) * S * logN \tag{3.5}$$

3.5 Preliminary Implementation and Analysis

In this example a sparse signal was taken, then compressive sampling was applied to represent it in the reduced length. It was reconstructed by the use of ll-minimization algorithm. The results confirms the ability of this technique to derive the actual signal from the much reduced dimension.

Let x be an sparse signal of length N (on DCT domain). To apply CS, the complete knowledge of the signal is not required, but a rough estimate on sparsity of the signal in some representation basis is required.

ϕ was choosen to be of random gaussian. The coherence between the DCT mtx and ϕ has to be computed. The reduced dimension can be computed from the above equation 3.5.

The graph shows the original, reduced and reconstructed signal.

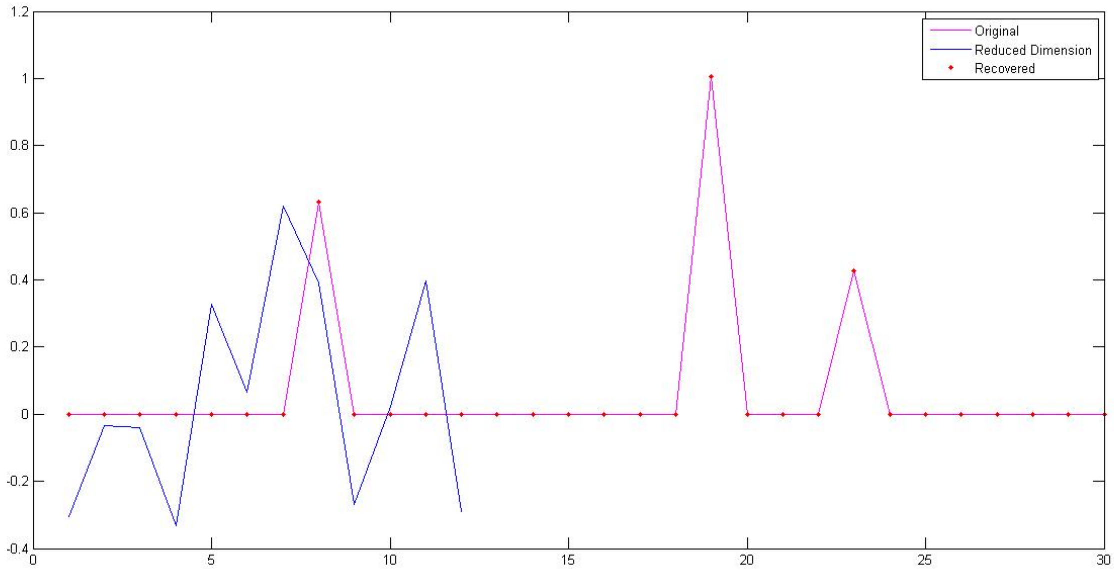


FIGURE 3.3: Preliminary Implementation of CS

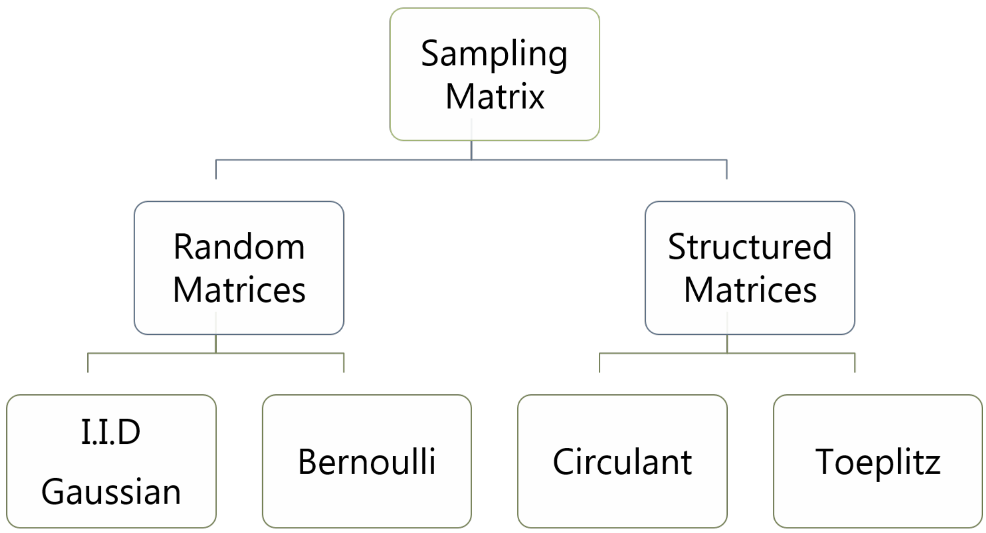
Analysis

- From the graph it is clear that, sparse signal in the representation domain got distributed in the reduced domain. This shows an incoherence between the representation and sampling basis system.
- As random sampling is adopted, for the same signal (at different instance)the nature of distribution may be different.

3.6 Types of Sampling Matrix

In compressive sampling, the widely preferred sampling matrix are random matrices (like i.i.d gaussian and bernoulli). These matrices satisfy restricted isometric property and they are largely incoherent with any fixed basis. But, it suffers from completely unstructured nature and costly to implement in hardware. Construction of similar random matrices at the reconstruction site is not possible, so during the transmission of compressed data, the complete information of sampling matrix also have to be sent. This puts additional burden on the data rate(bandwidth problem).

FIGURE 3.4: Types of sampling matrices



3.6.1 Structured Matrices

In order to reduce the computational burden and hardware complexities involved with random matrices, structured matrices are introduced. Toeplitz and Circulant matrices are proven to satisfy restricted isometric property. But the number of measurement required with structured matrices grows as the square of sparsity of the signal. Still, researches are going on in reducing and optimizing the exact number of measurements required for structured matrices[5].

Advantages of Structured matrices over Random matrices

- It make use of fewer independent random variable and can be constructed with less number of details/properties.
- Multiplication with Toeplitz or Circulant matrices can be implemented with FFT algorithm.
- Support block based processing

4 Compressed Measurement of SSIM

4.1 Introduction

This chapter elaborate the methods adopted to represent the data in the compressed domain and computation of SSIM given with the compressed data. Once the data got projected into a lower dimension it can be either processed or reconstructed. Here CM SSIM is done by reconstruction the data from the compressed domain given the knowledge of appropriate basis and some properties of the sampling matrix.

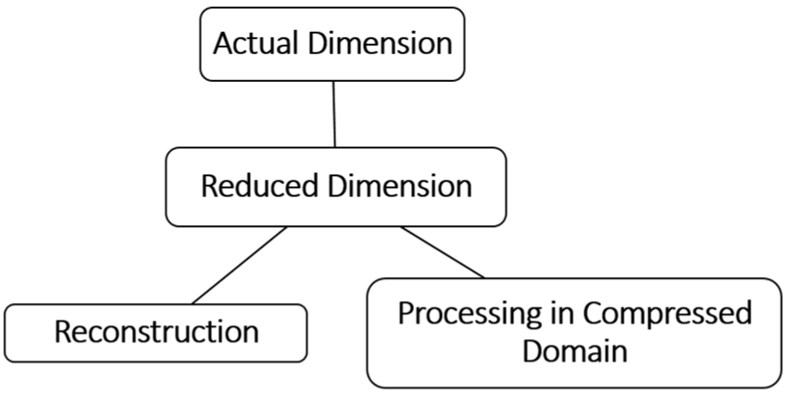


FIGURE 4.1: Compressed Processing

4.2 Implementation of CS

In compressed sampling, randomized samples were obtained based on the sparsity of the signal on the appropriate basis. Here in our work, CS was implemented considering sparsity of the signal in different dictionaries like discrete fourier transform, discrete cosine transform and wavelet.

Number of random measurements required for the signal is given by,

$$m \geq C * \mu^2(\psi, \phi) S log N \tag{4.1}$$

here S-sparsity; μ-Coherence; N-Actual Dimension; m-Reduced Dimension;

Minimum number of measurement for exact signal recovery grows linearly with respect to S. Before implementation, some prerequisite test have to be done to decide over number of random measurements and choice of sampling matrix.

4.2.1 Sparsity and Incoherence Test

Comparitive Analysis of Sparsity This test is done to estimate the capability of the dictionary to sparsify any given image. Here some images were randomly choosen from LIVE Database and transformed to DFT, DCT and Wavelet dictionaries. Comparitive analysis were done to have the rough estimate over the sparsity.

The sparsity was measured in terms of percentage of coefficients required to represent an image. From the results it can be inferred that compression capability of DCT and DWT are superior to DFT. The number of measurements required for CM SSIM DFT should be more than CM SSIM DCT & DWT.

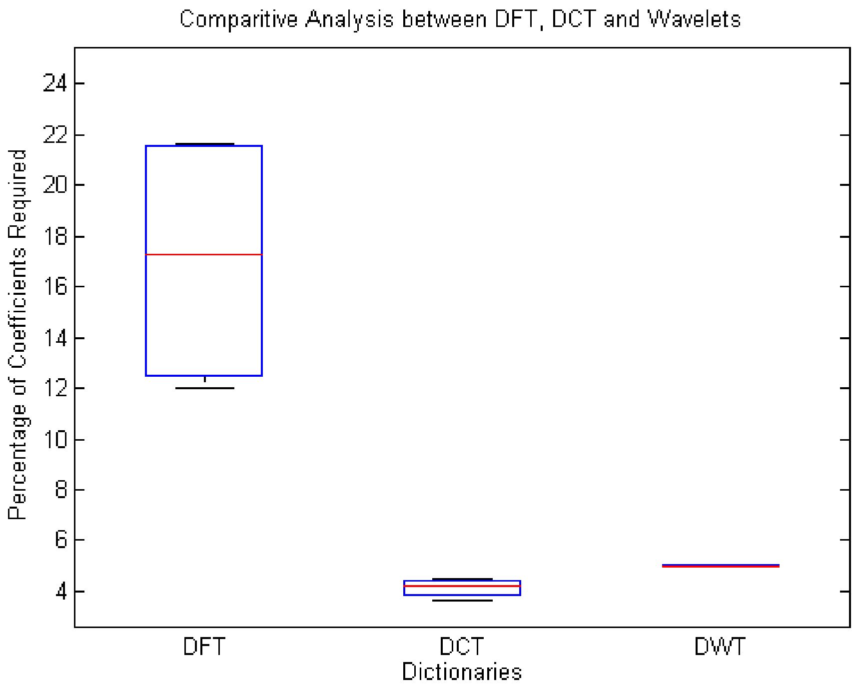


FIGURE 4.2: Sparsity analysis between DFT, DCT and DWT

Incoherence In this experiment, coherence value was computed between randomly generated gaussian matrices and circulant matrices with all the dictionary basis. The random matrices generated are of type i.i.d gaussian and structured matrices are of random ciruclant matrices.

From the result 4.3 and 4.4 it is clear that DWT and DCT exhibit good incoherence with the structured matrices. And for the DFT based basis, random gaussian matrices are preffered.

TABLE 4.1: Sampling Matrix Selection

Dictionary	Preferred Sampling Matrix
DCT, DWT	Random Ciculant Matrices
DFT	Random Matrices (i.i.d Gaussian/ Bernoulli)

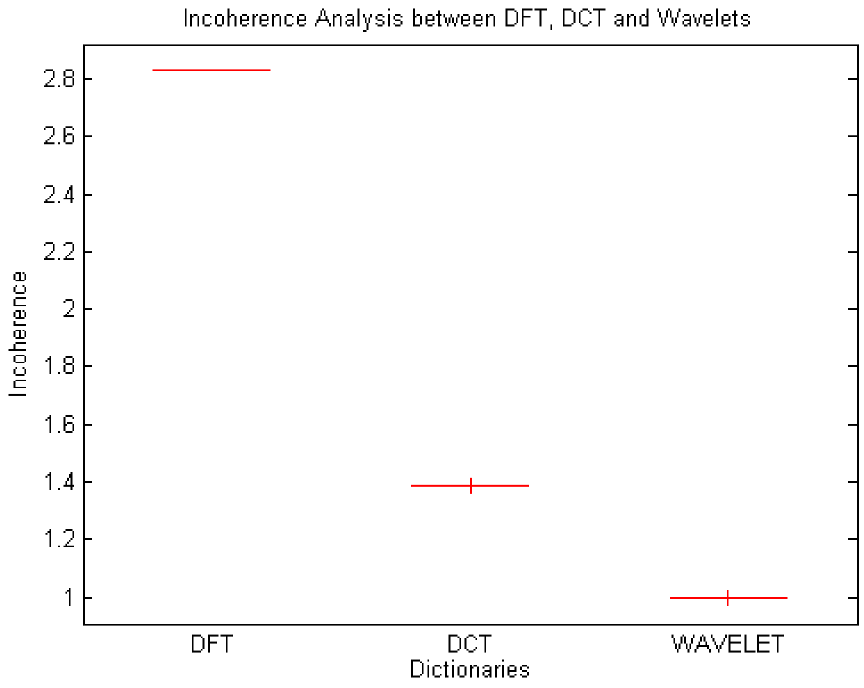


FIGURE 4.3: Coherence analysis(Random Circulant Matrices)

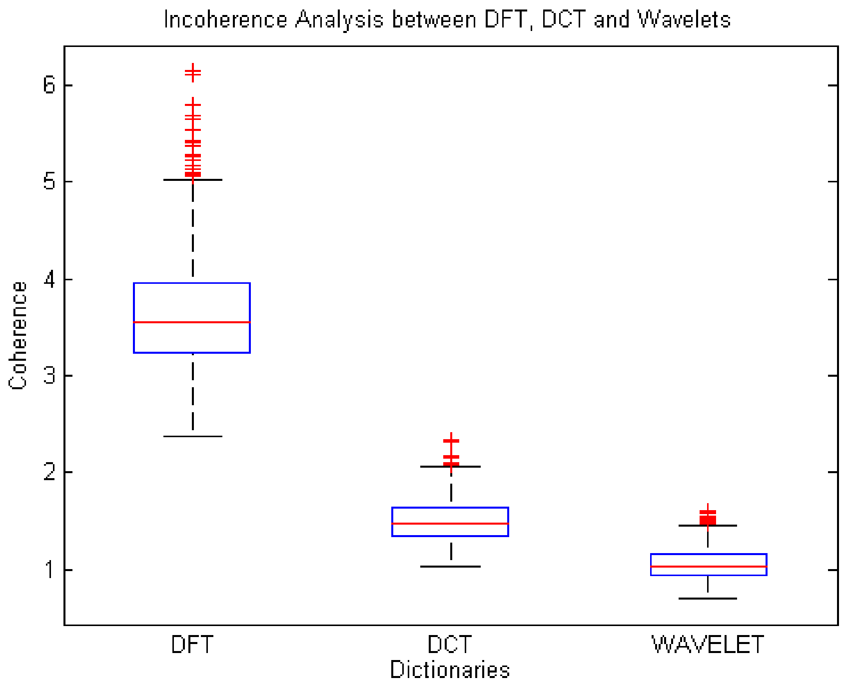


FIGURE 4.4: Coherence analysis (Random Gaussian Matrices)

4.3 CM SSIM

CM SSIM can be computed from compressed version of reference and test image. The method adopted is to reconstruct the data from the reduced dimension and compute SSIM based on dictionary coefficients. As compressed data are not available, test images were taken and projected to lower dimension by CS theory(assuming sparse at different dictionary). In the below, the techniques adopted to represent image in lower dimension and reconstruction are provided. The possibilities of processing in the compressed domain is also mentioned.

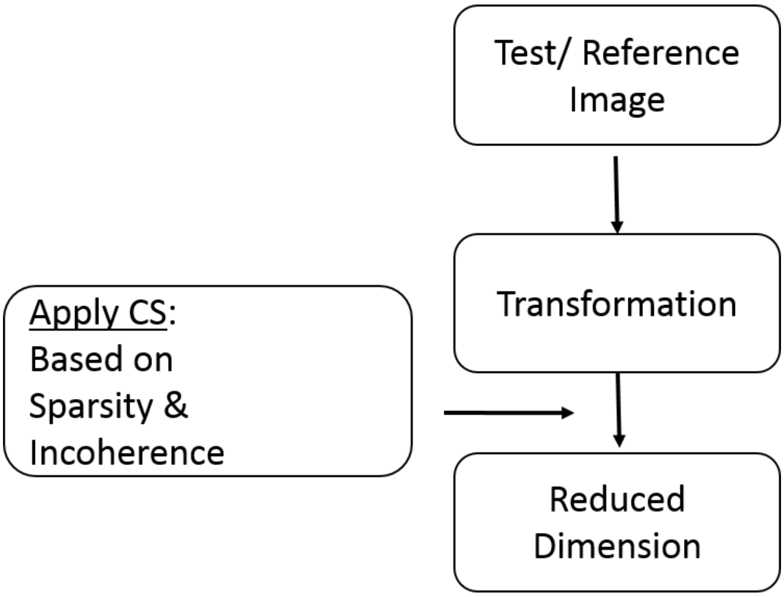


FIGURE 4.5: Projection: To Reduced Dimension

4.3.1 Representation in Reduced Dimension

This involves projecting a high dimension data(redundant data) into a much lower dimension through random sampling. Knowledge of sparsity of the data in dictionary space and incoherence value are required to fix the dimension of reduced representation.

A test/reference image is taken and transformed into appropriate basis system. Now depends on the sparsity of an image in that basis and coherence value with measurement matrix, it got projected into a reduced dimension. In practical system, this process is done by random sampling (ex. DMD based sensors), which directly acquire the compressed version of an image.

4.3.2 Processing in Compressed Domain

Manipulating data in the compressed domain has many advantages. Many practical application involves only detection and processing of data rather than complete reconstruction. In application like face/language detection, or RADAR, the interest lies in processing the signal and identifying patterns. Here, algorithms can be developed to find patterns based on compressed data. The problems in processing compressed data is that, the compressed domain is a space formed by randomly generated basis and thereby it is difficult to define statistical parameters associated with it (i.e. with every random acquisition the basis changes).

4.3.3 Reconstruction

Though images are represented and processed in compressed domain, reconstruction have to be done for human perception (ex. broadcasting).

The reconstruction is done on the reduced version of an test/reference image given with the knowledge of representation and measurement basis system. L1 norm minimizers or convex optimizers are used in the reconstruction process. The optimizers look for sparsest solution among the infinite solution set. The reconstructed sparse signal is in representation domain, and by using the table 2.2, CM SSIM can be computed.

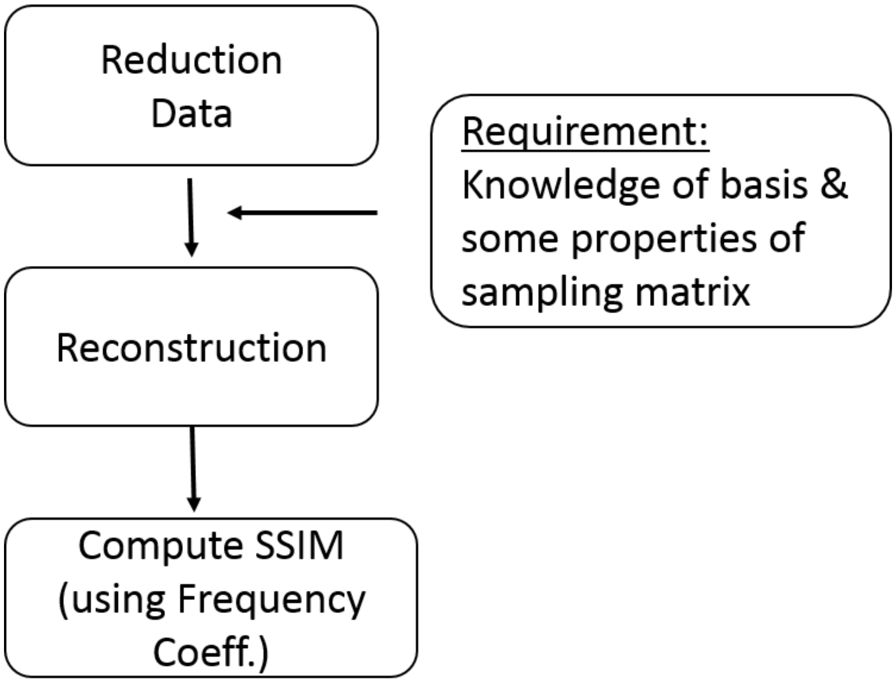


Figure 4.6: CM SSIM

Requirement for reconstruction

- The dimension of the space changes with respect to the representation basis. In case, if the representation basis is not available, then blind estimations have to be carried out.
- As random sampling protocol is adopted, complete knowledge or some properties of the measurement matrix is required for reconstruction.

4.4 CM SSIM MATLAB GUI

A graphical user interface for the CM SSIM was made. User input to the GUI are test and reference image. The GUI is provided with an option to compute SSIM in dictionary space with or without the application of CS. In case of CM SSIM, options are provided to choose for the type of sampling matrix.

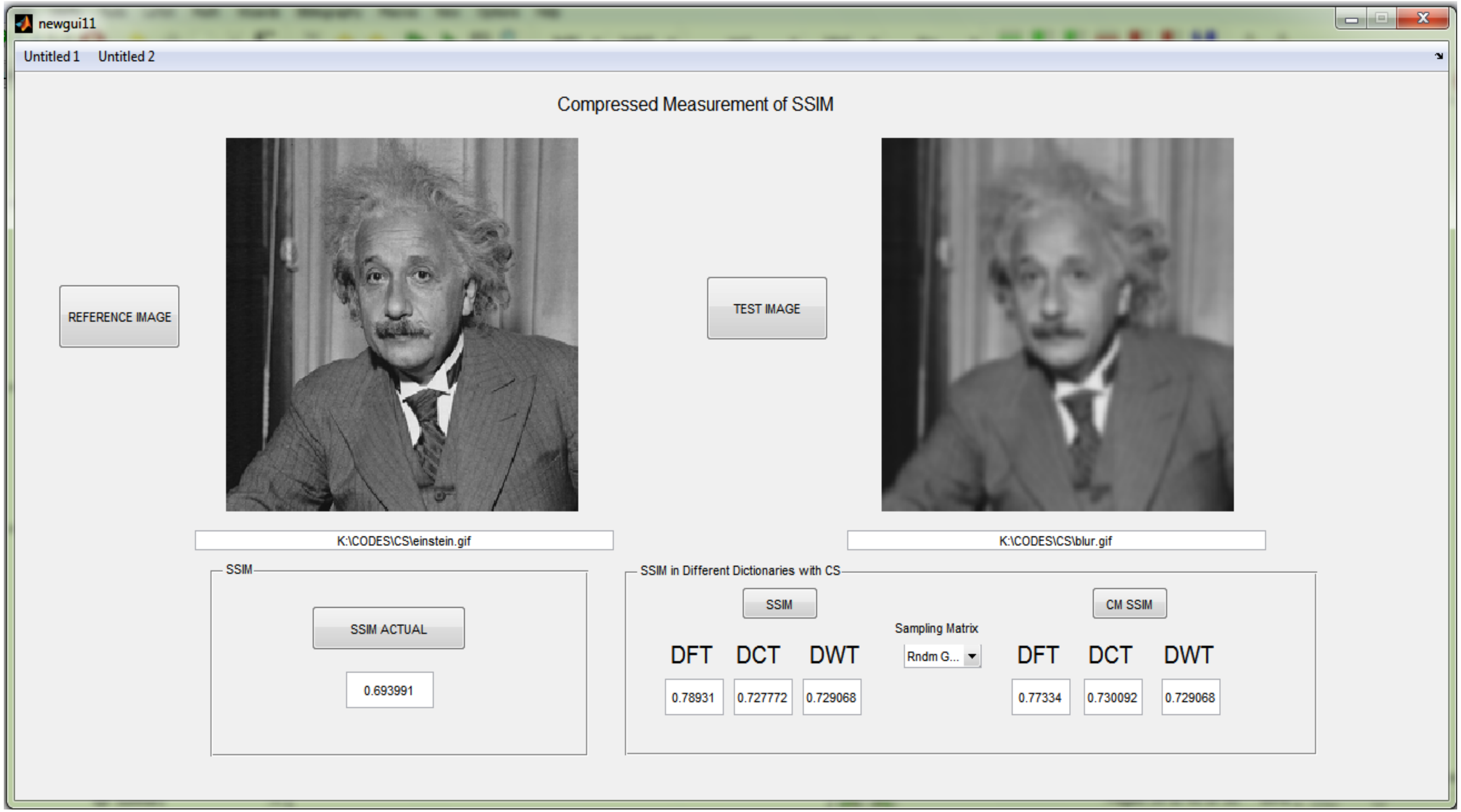


FIGURE 4.7: MATLAB GUI

5 Correlation Analysis

A new algorithm was proposed to compute the compressed measurement of Structural Similarity Index. To validate the performance and efficiency, the CM SSIM have to compared with standard SSIM algorithm and DMOS (subjective score). The accuracy can be validated based on the correlation coefficients obtained. The correlation coefficients considered for analysis are Pearson, Spearman and Kendall tau.

5.0.1 Image database for testing

Images required for the test analysis were taken from LIVE Database Release2[15]. The database have 29 reference image and their five different set of distortion. They are

- Guassian Blur
- White Noise
- Fast Fading
- Jpeg
- Jp2k

This open source database provides orignial and its distorted images in a seperate folder according to its type and distortion of different levels. Each distorted image has been provided with a quality score called differential mean opinion score(DMOS).This was computed from the various qualitative score given by group of people.

5.1 Correlation Test

5.1.1 Testing Parameters

While testing we calculate the correlation value between the DMOS value and the value obtained from actual SSIM and proposed CM-SSIM. The test was done for three different correlation factors. The result close to one shows the perfect relationship between the existing and proposed work.

5.1.2 Pearson Correlation

Pearson Correlation is a measure of linear association between two variables and is deonted by ρ.

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \tag{5.1}$$

It takes values from -1 to +1. The zero correlation indicates no association. The value greater than zero indicates a positive association that is if one tends to increase so does the value of other one. The values less than zero indiactes negative association that is if one tends to increase the other decreases. Mathematically it is given by ratio of covariance of two variable to product of their standard deviations.

The pearson coefficient was computed between the DMOS with the SSIM and CM SSIM to measure the linear association. DMOS score increases with decrease in image quality, whereas CM SSIM increases with increases in image quality. So values close to -1 represent a perfect linear relationship between DMOS and CMSSIM.

5.1.3 Spearman Correlation

It measures the strength of linear association between the ranked variables. In other words, it is a statistical measure of the monolithic relationship between paired data. The variables X_i and Y_i are converted to their ranks and correlation can be computed by

$$r_s = \frac{\sum_i^n((x_i) - \overline{(x_i)})((y_i) - \overline{(y_i)})}{\sqrt{\sum_i((x_i) - \overline{(x_i)})^2((y_i) - \overline{(y_i)})^2}} \tag{5.2}$$

- $r_s > 0$ indicates a positive monolithic association between two variables.
- $r_s < 0$ indicates a negative monolithic association between two variables.
- $r_s = 0$ indicates no association between two variables.

The correlation results close to -1 indicates perfect monolithic accuracy between the developed algorithm and the existing algorithm.

5.1.4 Kendall Tau Correlation

It is a non parametric measure of correlation between two ranked variables. Let $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$ be a set of observation of the joint random varibles X and Y respectively. Take any pair of observations $(x_i, y_i), (x_j, y_j)$ and they are said to be

- Concordant if $x_i > x_j$ and $y_i > y_j$ or $x_i < x_j$ and $y_i < y_j$
- Discordant if $x_i > x_j$ and $y_i < y_j$ or $x_i < x_j$ and $y_i > y_j$
- Neither Concordant or Discordant if $x_i = x_j$ and $y_i = y_j$

The formula is given by

$$\tau = \frac{(NumberofConcordantPairs) - (NumberofDiscordantPairs)}{1/2n(n-1)} \tag{5.3}$$

The coefficient must be in the range -1 $\leq \tau \leq 1$.

- Positive value (close to 1)indicates perfect agreement between the two rankings.
- Negative value suggests disagreement between the two rankings
- Zero indicates no agreement

BoxPlot It is a graphical representation of a data points of an variable. This plot give a better visual understanding over the data points deviation about its median. The plot is preseneted in terms of rectangular boxes and each represents a column vector. The median is represented by the central mark, and 25th and 75th percentiles are given by the edges. The line passing through the box is called as whiskers, the end points of the whiskers represent the extreme points of data-set excluding any outliers if present. The outliers are the values which are about twice the standard deviation from the mean of the data set.

5.2 Results

Correlation analysis was between DMOS value with actual SSIM and SSIM in DCT, DFT and DWT domain. From the results obtained it is concluded that there exist a good relationship between proposed and actual work.

5.2.1 Boxplots for different correlation factors

To get an better understanding over distribution of data points, box plots for different correlation coefficients were plotted.

Pearson Correlation

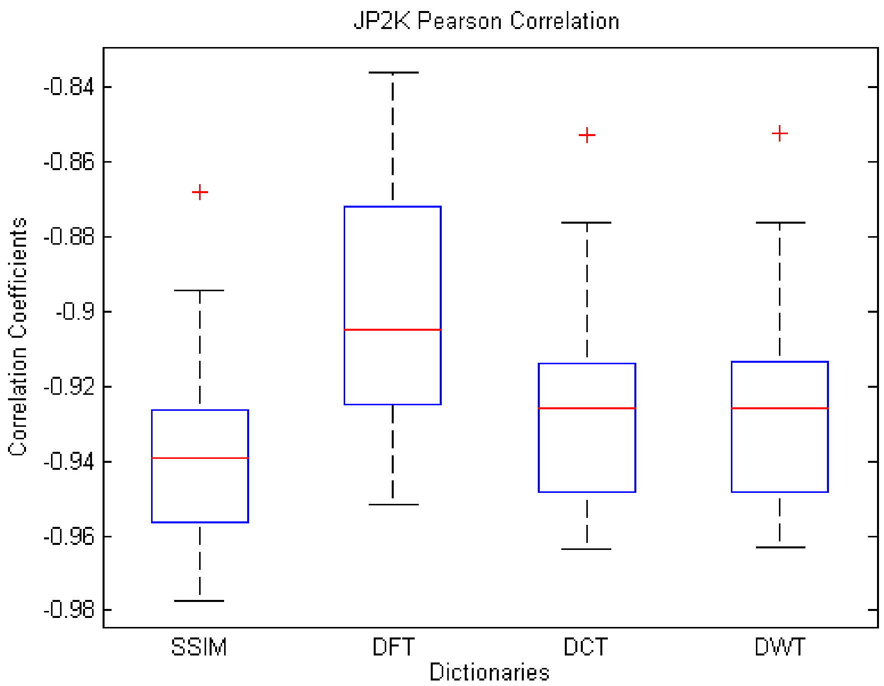


FIGURE 5.1: Box plot showing Pearson Correlation between DMOS with SSIM and CM-SSIM (DFT,DCT and DWT) for Live Image Database

Spearman Correlation

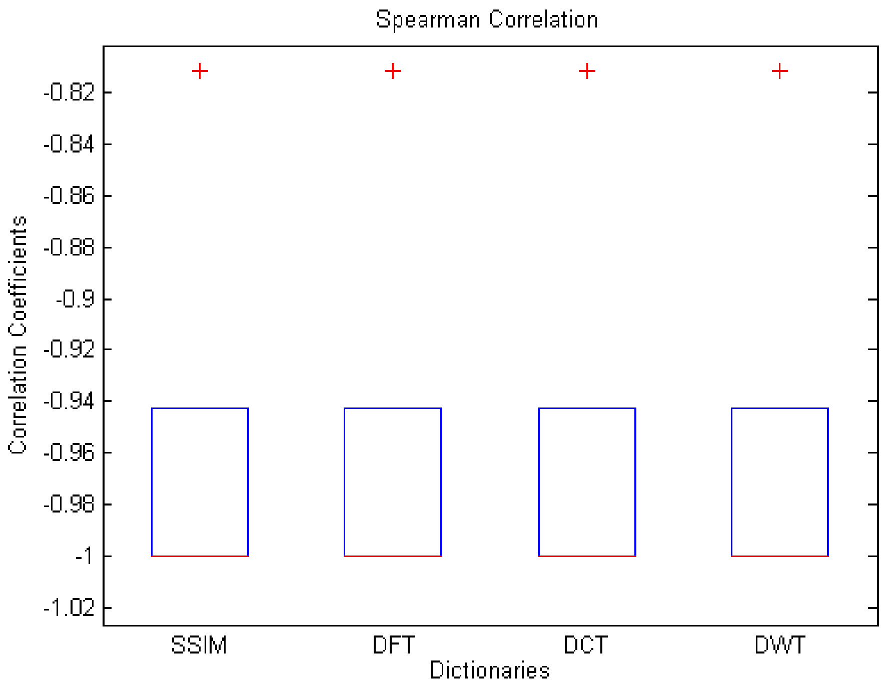


FIGURE 5.2: Box plot showing Spearman Correlation between DMOS with SSIM and CM-SSIM (DFT,DCT and DWT)for Live Image Database

Kendall Correlation

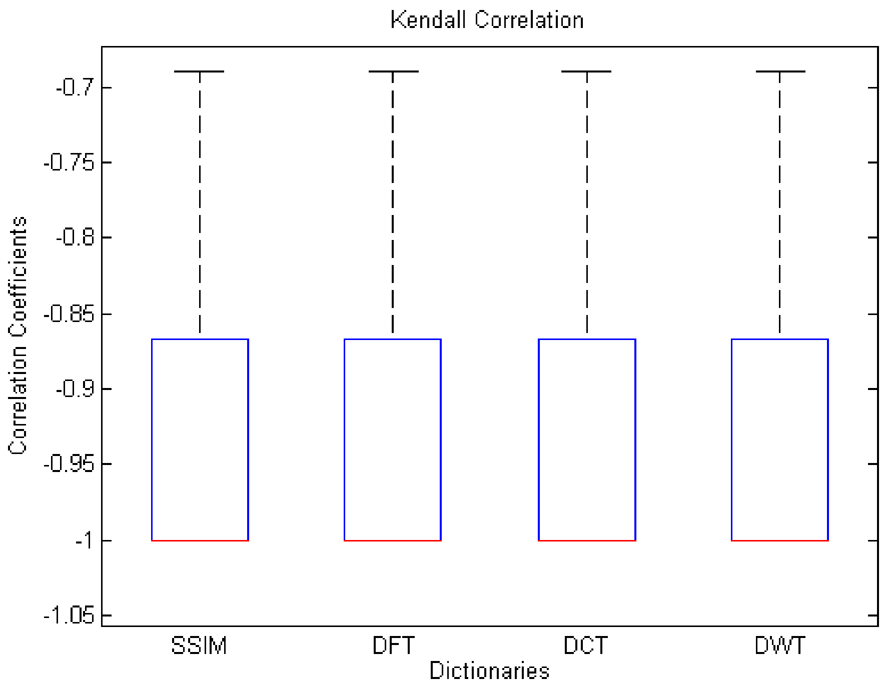


FIGURE 5.3: Box plot showing kendall correlation between DMOS with SSIM and CM-SSIM (DFT,DCT and DWT) for Live Image Database

From the box plot results , it can be inferred the variations in correlation value with DMOS are same for both SSIM and CM SSIM. This validates that CM-SSIM would be preferable for computation of quality index.

5.2.2 Summary

All the statistical analysis carried out to justify the performance measure of the proposed algorithm were described. The obtained correlation results were presented using tables and box plots to have better understanding and were analyzed.

6 Conclusion

6.1 Conclusion

From the results and analysis, we can conclude that the proposed algorithm is useful for computing SSIM in the Fourier, Cosine and Wavelet dictionaries. And this is made more effective by the implementation of compressive sampling. In overall, for future real time applications, proposed algorithm would be preferable in terms of reduced memory and time usage.

6.2 Future Work

Even though compressive sampling is well proven mathematically, its hardware realization involves lot of difficulties. And future scope of this project is to make this algorithm more suitable for real time hardware implementations. And another scope is to develop a blind reconstruction algorithm given with the compressed data.

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